Refinement in a Separation Context

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Introduction

- Hoare did data refinement for imperative programs

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- As usual dangling pointers are the problem
- Linguistic approaches haven't worked

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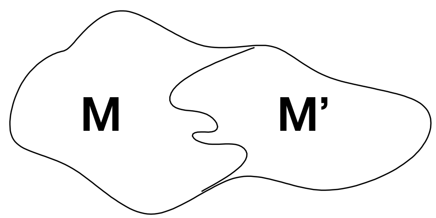
$$M{*}M' = \{(s,h) \mid \exists h_0, h_1.\ h_0 \# h_1 \wedge h = h_0{*}h_1 \wedge (s,h_0) \in M \wedge (s,h_1) \in M'\}.$$



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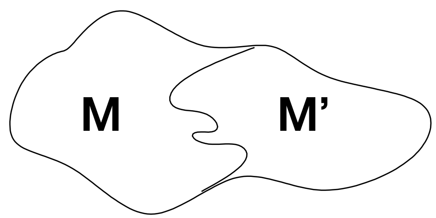

Let $t \subseteq (S \times H) \times (S \times H) \uplus \{wrong\}$. The relation $M \subseteq S \times H$ is **preserved** by relation t if for all $(s,h),(s',h')$, $(s,h) \in M$ and $(s,h)[t](s',h')$, imply $(s',h') \in M$.

Separation Context

$$c_{user} ::= \ \mathbf{oper}_i,\ i \in I \mid \mathbf{skip} \mid x := e \mid x := [e] \mid [e] := e \mid c_1; c_2$$
$$\mid \mathbf{if}\ e\ \mathbf{then}\ c_1\ \mathbf{else}\ c_2 \mid \mathbf{while}\ e\ \mathbf{do}\ c$$

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Let $M \subseteq S \times H$ be a precise unary relation, and for $i \in I$ let $oper_i$ preserve relation $M * \mathsf{T}$. A program c is a **unary separation context** for M and $(oper_i)_{i \in I}$ if for all executions and all $(s, h) \in M * \mathsf{T}$ $c, s, h \not\rightsquigarrow av$ and $c, s, h \not\rightsquigarrow wrong$.

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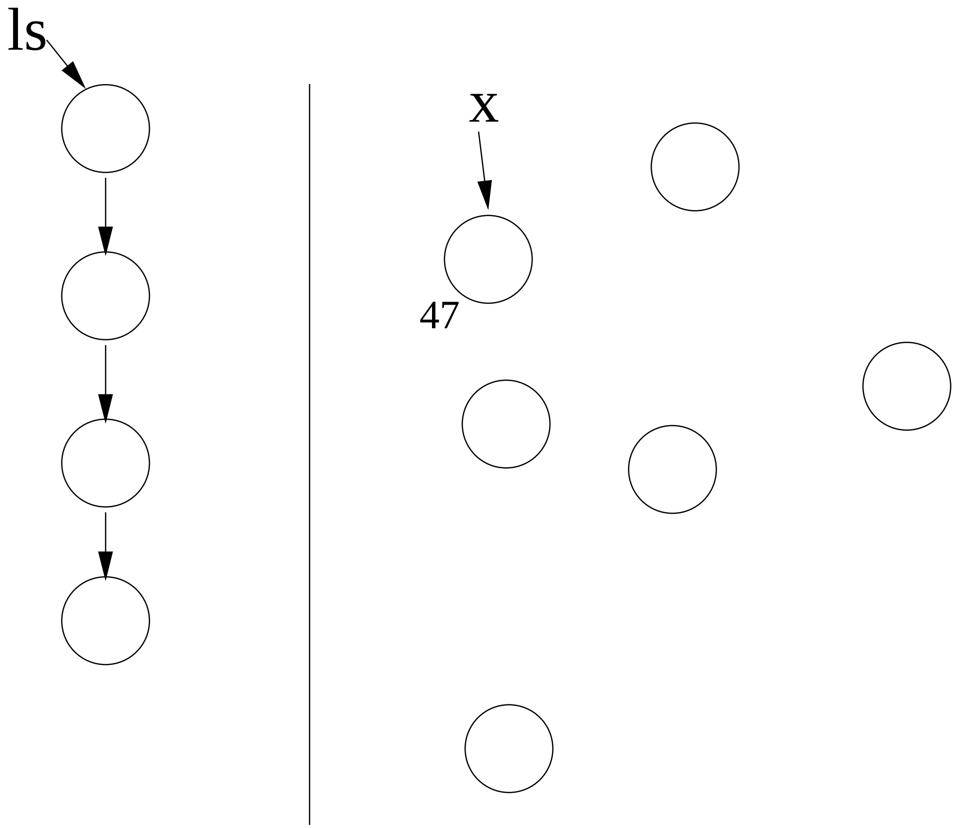
Let $M \subseteq S \times H$ be a precise relation, and for $(i \in I)$ let $oper_i$ preserve $M * \mathsf{T}$, and let c be a separation context for M and $(oper_i)_{i \in I}$. If $(s, h) \in M * \mathsf{T}$, and $c, s, h \rightsquigarrow s', h'$, then $(s', h') \in M * \mathsf{T}$.

Separation context



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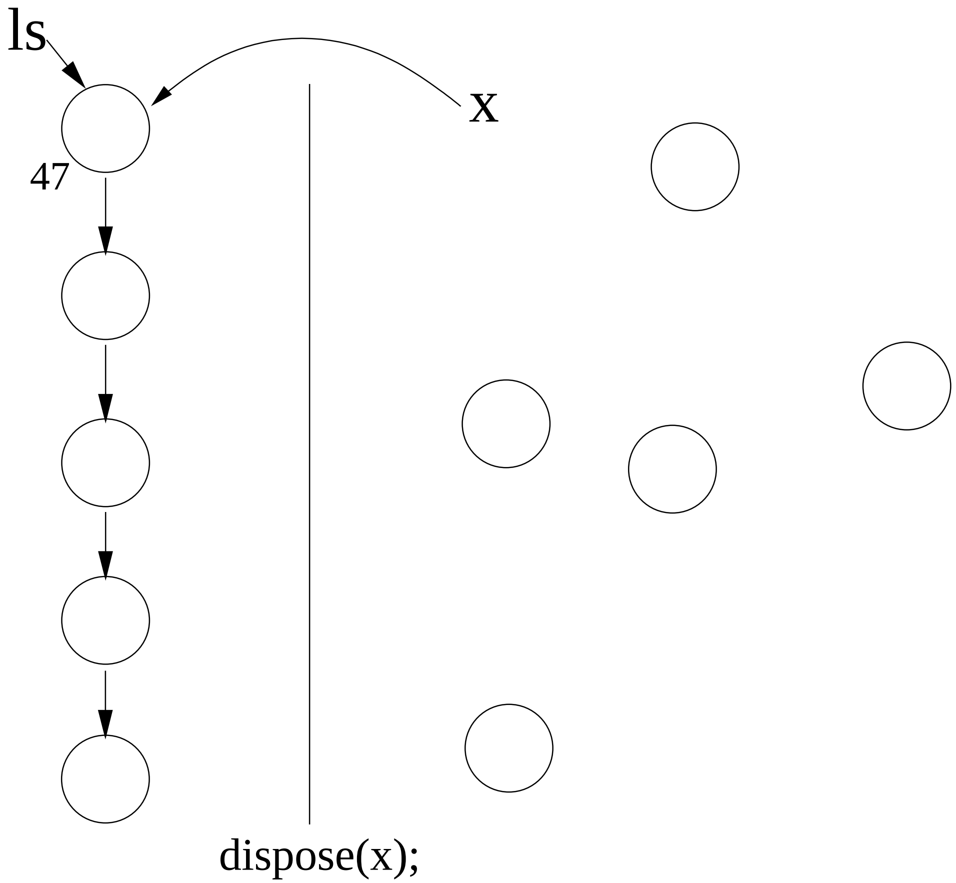

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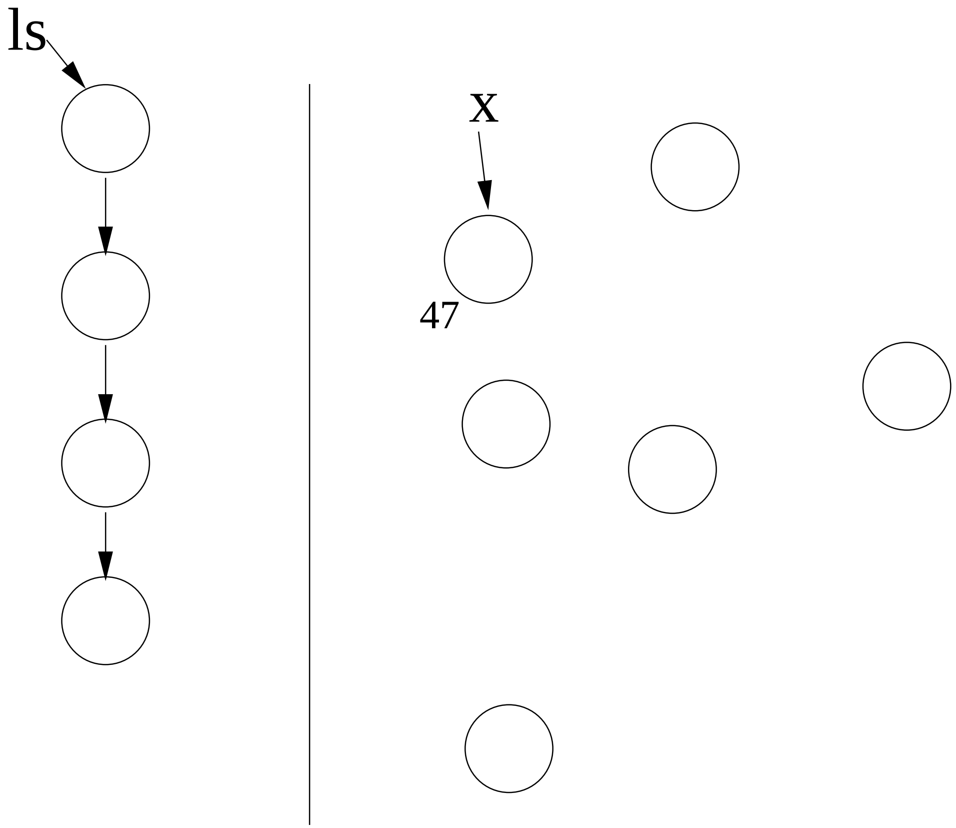
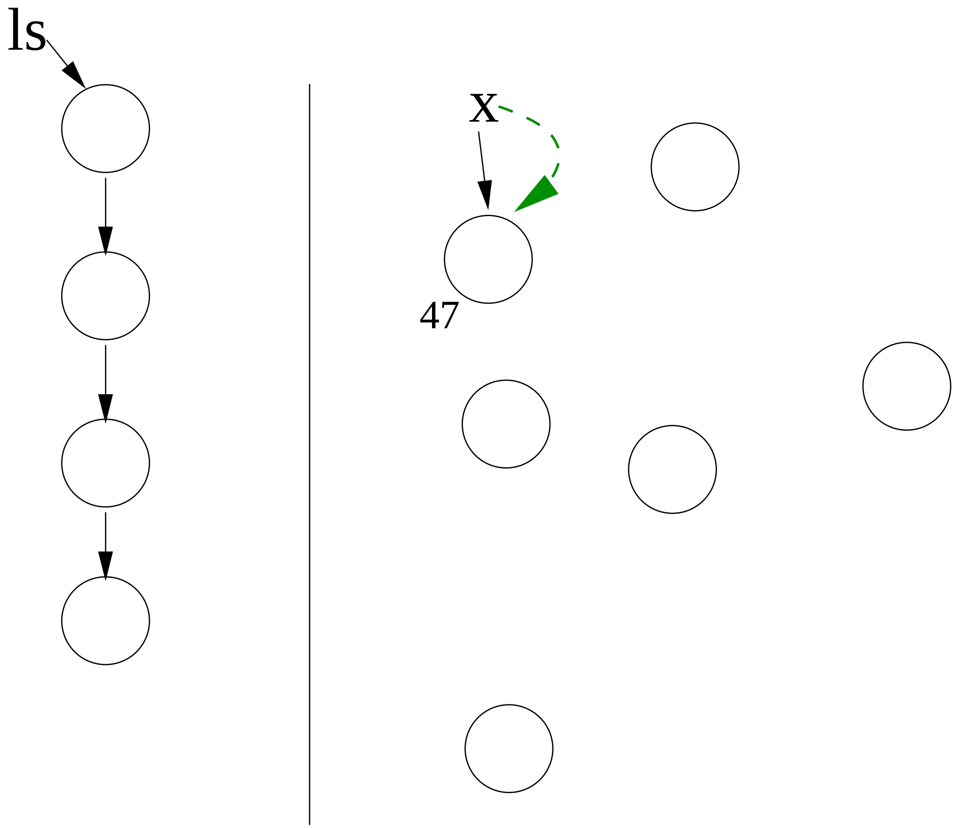


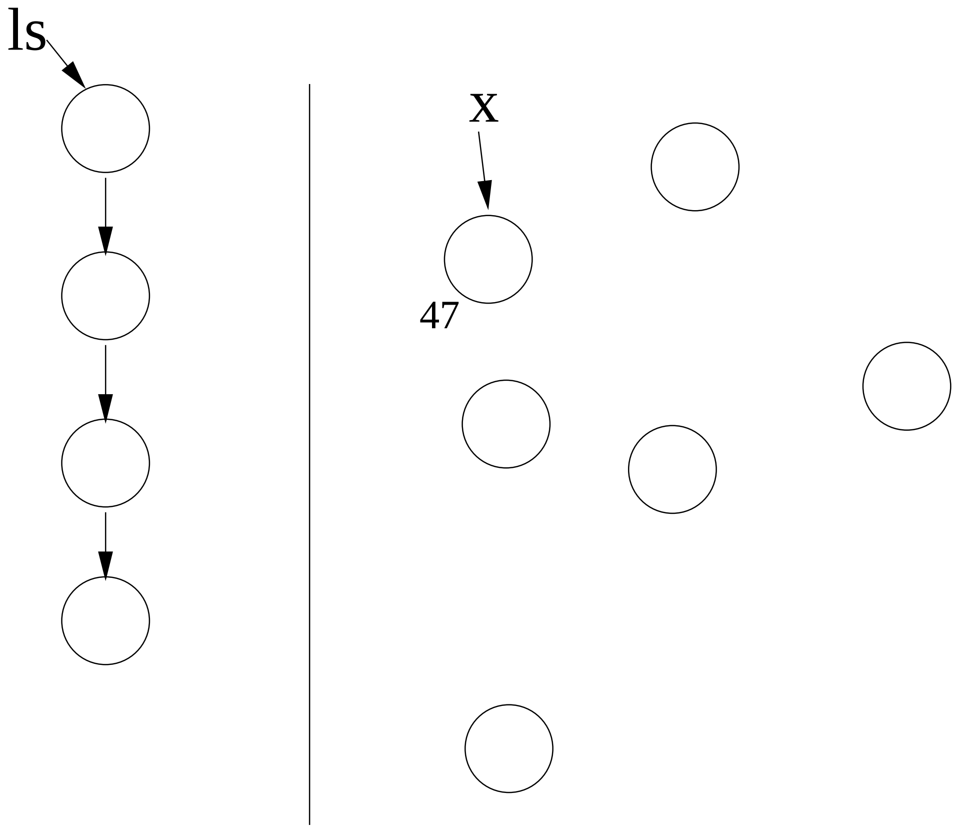
dispose(x); x=new();

Separation context



dispose(x); x=new(); y=[x]

Non-separation context



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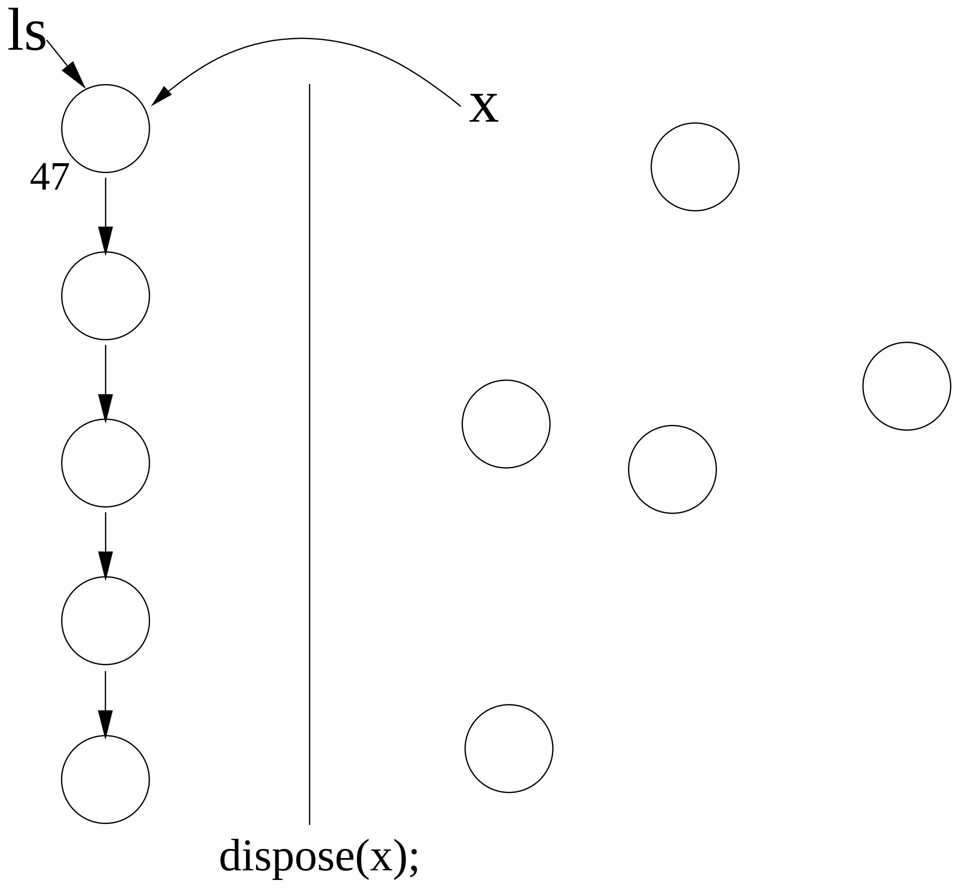

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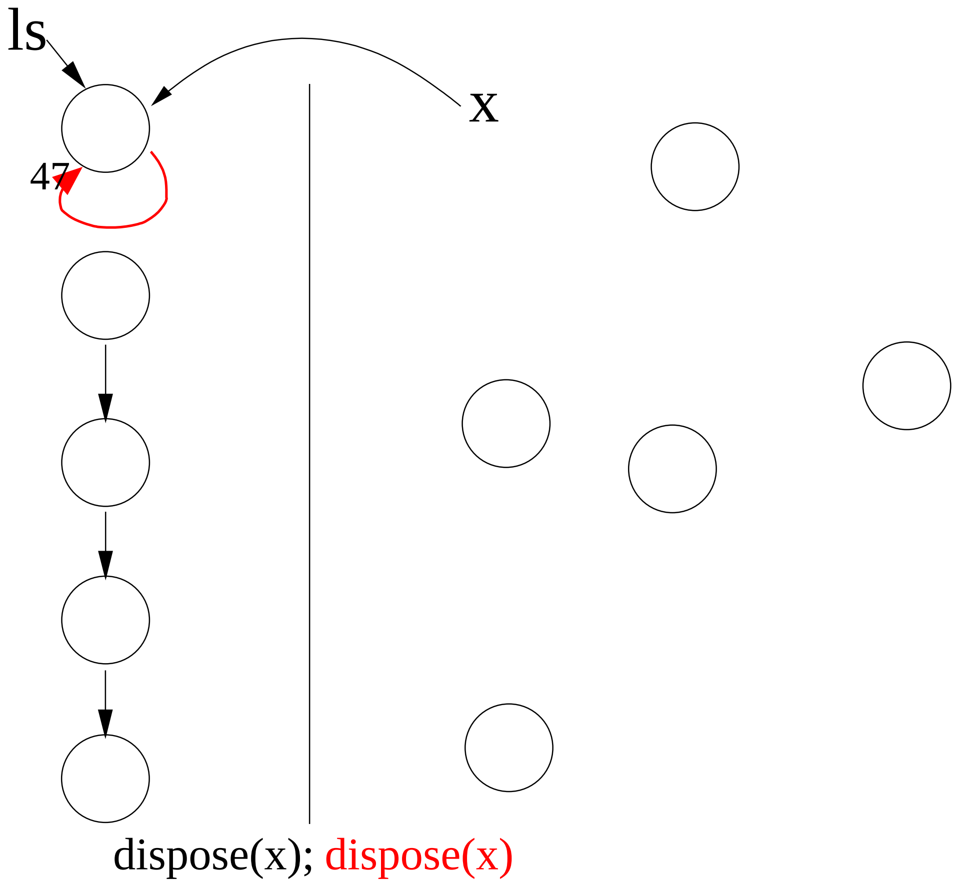

Binary Relations for Refinement

We say that binary relation R is **precise**, if each of its two projections on the set of states is precise.

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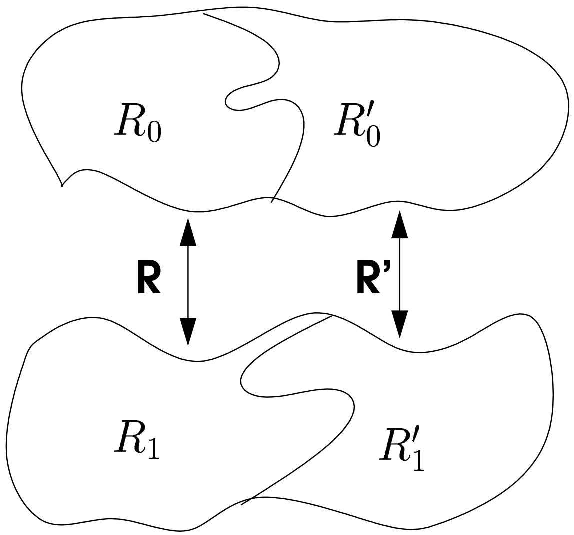
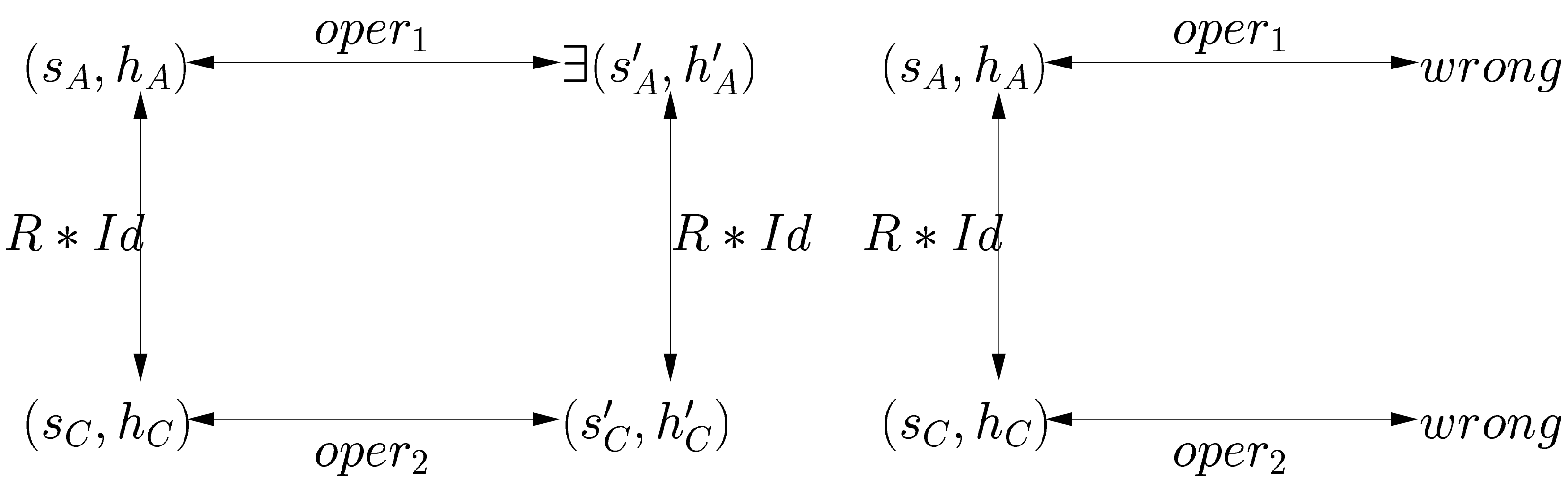
Separating conjunction of binary relations



Refinement



$oper_1$
(s_A, h_A)
$\exists(s'_A, h'_A)$
$R * Id$
$R * Id$
(s_C, h_C)
(s'_C, h'_C)
$oper_2$
$oper_1$
(s_A, h_A)
$wrong$
$R * Id$
(s_C, h_C)
$wrong$
$oper_2$

The Result

- A separation context for the abstract data type is a separation context for all its refinements

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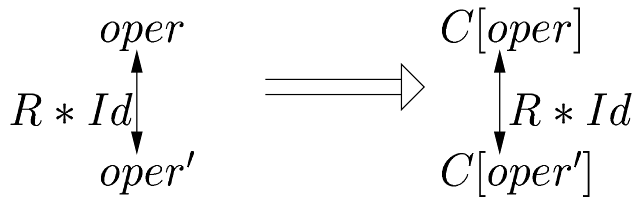
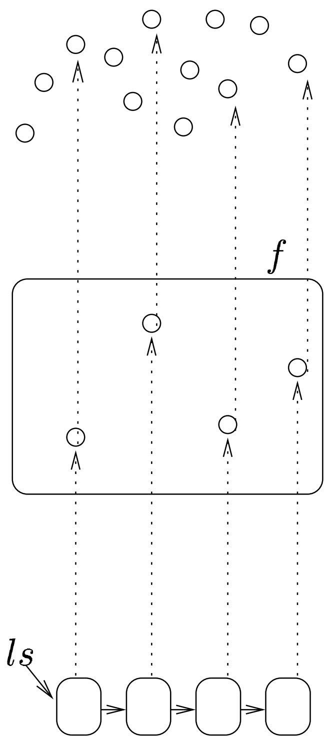
- Sepration contexts preserve $R * \mathsf{Id}$



Example - $new()$ and $dispose()$



Abstract – Magic

RI: emp

Intermediate – Set

RI: $\forall_* p \in f. p \mapsto \_, \_$

Concrete – List

RI: $list(\alpha, ls.nil)$

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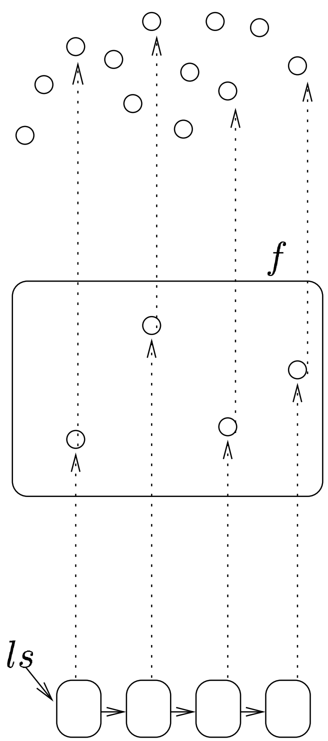

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$$R_1 = \{((s_A, h_A), (s_C, h_C)) \mid s_A, h_A \Vdash emp \wedge (s_C, h_C \Vdash \forall_* p \in f.\ p \mapsto \_, \_)\}$$

Future Work

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- We would like to have a logic